# Memory Based Answers \& Solutions 

Time : 3 hrs.

# JEE (Main)-2024 (Online) Phase-1 

(Physics, Chemistry and Mathematics)

## IMPORTANT INSTRUCTIONS:

(1) The test is of $\mathbf{3}$ hours duration.
(2) This test paper consists of 90 questions. Each subject (PCM) has 30 questions. The maximum marks are 300.
(3) This question paper contains Three Parts. Part-A is Physics, Part-B is Chemistry and Part-C is Mathematics. Each part has only two sections: Section-A and Section-B.
(4) Section - A : Attempt all questions.
(5) Section - B : Attempt any 05 questions out of 10 Questions.
(6) Section-A (01-20) contains 20 multiple choice questions which have only one correct answer. Each question carries $\mathbf{+ 4}$ marks for correct answer and $\mathbf{- 1}$ mark for wrong answer.
(7) Section-B(21-30) contains 10 Numerical value based questions. The answer to each question should be rounded off to the nearest integer. Each question carries $\boldsymbol{+ 4} \mathbf{~ m a r k s}$ for correct answer and -1 mark for wrong answer.

## PHYSICS

## SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which ONLY ONE is correct.

## Choose the correct answer:

1. A body of man 100 kg travelled 10 m before coming to rest. If $\mu=0.4$, work done against friction is (motion is happening on horizontal surface, take $g=10 \mathrm{~m} / \mathrm{s}^{2}$ )
(1) 4500 J
(2) 5000 J
(3) 4200 J
(4) 4000 J

## Answer (4)

Sol. $\frac{v^{2}}{2 a}=s \quad(a=\mu g)$
$v^{2}=2 \times \mu g \mathrm{~s}$
$V^{2}=2 \times(.4) \times 10 \times 10$
$v^{2}=80$
$w_{f}=\Delta k$
$=-\frac{1}{2} \times 100 \times 80$
$w_{f}=-4000$
2. If an object is having same weight at same distance above and below the surface of earth, find its distance from surface of earth.
(1) $\frac{R}{2}$
(2) $(\sqrt{5}-1) \frac{R}{2}$
(3) $(\sqrt{3}-1) \frac{R}{2}$
(4) $(\sqrt{5}-1) R$

## Answer (2)

Sol. $\frac{G M m}{(R+x)^{2}}=\frac{G M m(R-x)}{R^{3}}$
$\Rightarrow R^{3}=(R+x)^{2}(R-x)$
$\Rightarrow R^{3}=\left(R^{2}-x^{2}\right)(R+x)$
$\Rightarrow x^{2}+R x-R^{2}=0$
$\therefore x=\frac{-R \pm \sqrt{R+4 R^{2}}}{2}$
$x=\frac{(\sqrt{5}-1)}{2} R$
3. Consider the two statements :

Statement-1 : A capillary tube is first dipped in hot water and then dipped in cold water. The rise is higher in hot water.

Statement-2 : Capillary tube is first dipped in cold water and then dipped in hot water. The rise is higher in cold water.
(1) Statement 1 is true and statement- 2 is false
(2) Statement- 1 is false and statement-2 is true
(3) Both statements are true
(4) Both statements are false

## Answer (2)

Sol. $h=\frac{2 S \cos \theta}{\rho g R}$
as $T \uparrow, S \downarrow$
The correct answer is Option (2).
4. If a particle starting from rest having constant acceleration covers distance $S_{1}$ in first ( $p-1$ ) seconds and $S_{2}$ in first $p$ seconds, then determine time for which displacement is $S_{1}+S_{2}$
(1) $\sqrt{2 p^{2}+1-2 p}$
(2) $\sqrt{2 p^{2}+1+2 p}$
(3) $\sqrt{(p-1)^{2}-p}$
(4) $2 p$

Answer (1)
Sol. $S_{1}=\frac{1}{2} a(p-1)^{2}$
$S_{2}=\frac{1}{2} a p^{2}$
$S_{1}+S_{2}=\frac{1}{2} a\left[(p-1)^{2}+p^{2}\right]=\frac{1}{2} a t^{2}$
$t=\sqrt{2 p^{2}+1-2 p}$
5. de-Broglie wavelength of a proton and an electron is same. The ratio of kinetic energy of electron to that of proton is
(1) 1
(2) 1835
(3) $\frac{1}{1867}$
(4) 933.5

Answer (2)
Sol. $\frac{h}{p_{1}}=\frac{h}{p_{2}}$

$$
\Rightarrow \quad \sqrt{2 m_{1} k_{1}}=\sqrt{2 m_{2} k_{2}}
$$

$$
\Rightarrow \frac{k_{2}}{k_{1}}=\frac{m_{1}}{m_{2}}=1835
$$

6. If ratio of centripetal acceleration of two particles moving on the same path is $3: 4$. Find the ratio of their tangential velocities.
(1) $2: \sqrt{3}$
(2) $\sqrt{3}: 2$
(3) $\sqrt{3}: 1$
(4) $\sqrt{2}: 1$

## Answer (2)

Sol. $a_{c}=\frac{v^{2}}{r}, \frac{\left(a_{c}\right)_{1}}{\left(a_{c}\right)_{2}}=\left(\frac{v_{1}}{v_{2}}\right)^{2}$

$$
\frac{3}{4}=\left(\frac{v_{1}}{v_{2}}\right)^{2} \rightarrow \frac{v_{1}}{v_{2}}=\sqrt{3}: 2
$$

7. A capacitor having capacitance of $100 \mu \mathrm{~F}$ is charged with a potential difference of 12 V is connected to an inductor of inductance 10 mH . Find the maximum current through the inductor.
(1) 2 A
(2) 1.6 A
(3) 2.4 A
(4) 1.2 A

## Answer (4)

Sol. $I=Q_{0 . \omega}$

$$
\begin{aligned}
& =\frac{C V}{\sqrt{L C}}=V \sqrt{\frac{C}{L}} \\
& =12 \sqrt{\frac{100 \times 10^{-6}}{10 \times 10^{-3}}} \\
& =1.2 \mathrm{~A}
\end{aligned}
$$

8. A square loop of resistance $16 \Omega$ is connected with battery of 9 V and internal resistance of $1 \Omega$. In steady state, find energy stored in capacitor of capacity $C=4$ $\mu F$ as shown (at steady state current divides symmetrically)

(1) $51.84 \mu \mathrm{~J}$
(2) $12.96 \mu \mathrm{~J}$
(3) $25.92 \mu \mathrm{~J}$
(4) $103.68 \mu \mathrm{~J}$

## Answer (3)

Sol. Equivalent circuit

$i=\frac{9}{4+1}=1.8 \mathrm{~A}$
$\Rightarrow \frac{i}{2}=0.9 \mathrm{~A}$

$\left(V_{P}-V_{Q}\right)=0.9 \times 6-0.9 \times 2$
$V_{C}=3.6 \mathrm{~V}$
$U=\frac{1}{2} C V^{2}=\frac{1}{2} \times 4 \times 3.6 \times 3.6 \mu \mathrm{~J}$
$=25.92 \mu \mathrm{~J}$
9. A gas undergoes a cyclic process $A B C A$ as shown. Find the work done by the gas for $A \longrightarrow B \longrightarrow C$.

(1) 1800 J
(2) 1200 J
(3) 3600 J
(4) 600 J

## Answer (2)

Sol. Work = Area

$$
\begin{aligned}
\Rightarrow \quad W & =\frac{1}{2} \times 600 \times 4 \\
& =1200 \mathrm{~J}
\end{aligned}
$$

10. If a biconvex lens of material of refractive index 1.5 has focal length 20 cm in air, then its focal length when it is submerged in a medium of refractive index 1.6 is
(1) -160 cm
(2) 160 cm
(3) 1.6 cm
(4) -16 cm

## Answer (1)

Sol. $\frac{1}{20}=(1.5-1)\left(\frac{2}{R}\right)$
$R=20 \mathrm{~cm}$
$\frac{1}{f^{\prime}}=\left(\frac{1.5}{1.6}-1\right)\left(\frac{2}{R}\right)$
$=\frac{-1}{16} \times \frac{2}{20}$
$f^{\prime}=-160 \mathrm{~cm}$
11. If electric current passing through a conductor varies with time as $I=I_{0}+\beta t$, where $I_{0}=20 \mathrm{~A}, \beta=3 \mathrm{~A} / \mathrm{s}$, then find charge flow through conductor in first 10 sec.
(1) 400 C
(2) 500 C
(3) 200 C
(4) 350 C

Answer (4)
Sol. $\Rightarrow d=\int I . d t=\int_{0}^{10}(20+3 t) d t$

$$
=(20 t)_{0}^{10}+3\left(\frac{t^{2}}{2}\right)_{0}^{10}=350 \mathrm{C}
$$

12. Consider a series of steps as shown. A ball is thrown from $O$. Find the minimum speed of directly jump to $5^{\text {th }}$ step.

(1) $5(\sqrt{2}+1) \mathrm{m} / \mathrm{s}$
(2) $5 \sqrt{2} \mathrm{~m} / \mathrm{s}$
(3) $5 \sqrt{\sqrt{2}+1} \mathrm{~m} / \mathrm{s}$
(4) $6 \sqrt{\sqrt{3}+1} \mathrm{~m} / \mathrm{s}$

## Answer (3)

Sol. $y=x \tan \theta-\frac{g x^{2}}{2 v^{2} \cos ^{2} \theta}$
$(2.5,2.5)$ must lie on this
$\Rightarrow 1=\tan \theta-\frac{g \times 2.5}{2 v^{2} \cos ^{2} \theta}$
$\Rightarrow \frac{25}{2 v^{2} \cos ^{2} \theta}=\tan \theta-1$
$\Rightarrow v^{2}=\frac{25}{2}\left\{\frac{1+\tan ^{2} \theta}{\tan \theta-1}\right\}$
$\Rightarrow v_{\text {min }}=5 \sqrt{\sqrt{2}+1}$
[Happens when $\tan \theta=\sqrt{2}+1$ ]
13. An electron is moving with speed of $1 \mathrm{~m} / \mathrm{s}$ at distance of 1 m from a large sheet of charge with density $\sigma \mathrm{C} / \mathrm{m}^{2}$. Find maximum value of $\sigma$ such that electron hit the sheet after 1 sec .

(mass of electron $9 \times 10^{-31} \mathrm{~kg}$, permittivity of free space $\varepsilon_{0}=9 \times 10^{-12} \mathrm{C}^{2} / \mathrm{Nm}^{2}$ )
(1) $4.05 \times 10^{-22} \mathrm{C} / \mathrm{m}^{2}$
(2) $8.10 \times 10^{-22} \mathrm{C} / \mathrm{m}^{2}$
(3) $4.05 \times 10^{24} \mathrm{C} / \mathrm{m}^{2}$
(4) $2.02 \times 10^{-20} \mathrm{C} / \mathrm{m}^{2}$

## Answer (1)

Sol. For maximum value of $\sigma$, initially, electron must move away from plate.
$u t+\frac{1}{2} a t^{2}=s$
$t=1 \quad u=1 \mathrm{~m} / \mathrm{s} \quad s=-1 \mathrm{~m}$
$1 \times 1-\frac{1}{2} a \times 1^{2}=-1$
$\Rightarrow \quad a=4 \mathrm{~m} / \mathrm{s}^{2}$

$$
\frac{q E}{m}=4
$$

$\frac{q \sigma}{2 \varepsilon_{0} m}=4$

$$
\begin{aligned}
\sigma & =\frac{4 \times 2 \times 9 \times 10^{-12} \times 9 \times 10^{-31}}{1.6 \times 10^{-19}} \\
& =\frac{8 \times 81}{1.6} \times 10^{-24} \\
& =4.05 \times 10^{-22} \mathrm{C} / \mathrm{m}^{2}
\end{aligned}
$$

14. In the voltage regulator circuit shown below, the reverse breakdown voltage of zener diode is 3 V . Find the current through zener diode.

(1) 7 mA
(2) 1.5 mA
(3) 5.5 mA
(4) 10 mA

Answer (3)
Sol.
$i_{\text {battery }}=\frac{10-3}{1000}=7 \mathrm{~mA}$
${ }^{i} 2 \mathrm{k} \Omega=\frac{3}{2000}=1.5 \mathrm{~mA}$
$i_{z}=(7-1.5) \mathrm{mA}$
$=5.5 \mathrm{~mA}$
15. Consider the circuit shown. Galvanometer resistance is $10 \Omega$ and current through galvanometer is 3 mA . Find the resistance of shunt.

(1) $10^{-3} \Omega$
(2) $7.5 \times 10^{-3} \Omega$
(3) $6.75 \times 10^{-3} \Omega$
(4) $3.75 \times 10^{-3} \Omega$

Answer (4)

Sol. Since $G$ and $S$ are in parallel

$$
\begin{aligned}
& \Rightarrow V_{\mathrm{G}}=V_{\mathrm{S}} \\
& \Rightarrow 3 \mathrm{~mA} \times 10=8 \mathrm{~A} \times R_{\mathrm{S}} \\
& \Rightarrow R_{\mathrm{S}}=3.75 \mathrm{~m} \Omega
\end{aligned}
$$

16. A particle executing simple harmonic motion along $x$-axis, with amplitude $A$, about origin. If ratio of kinetic energy and total energy at $x=\frac{A}{3}$ is
(1) $\frac{8}{9}$
(2) $\frac{7}{8}$
(3) $\frac{1}{9}$
(4) $\frac{1}{8}$

## Answer (1)

Sol. $K E=\frac{1}{2} m \omega^{2}\left(A^{2}-n^{2}\right)$

$$
T E=\frac{1}{2} m \omega^{2} A^{2}
$$

$$
\frac{K E}{T E}=\frac{A^{2}-n^{2}}{A^{2}}=\frac{1-\frac{1}{9}}{1}=\frac{8}{9}
$$

17. 
18. 
19. 
20. 

## SECTION - B

Numerical Value Type Questions: This section contains 10 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.
21. A solid sphere of radius $4 a$ with centre at origin. Two charge, $-2 q$ at $(-5 a, 0)$ and $5 q$ at $(3 a, 0)$ is placed. Flux through sphere is $\frac{x q}{\varepsilon_{0}}$. Find $x$

Answer (5)

Sol.


From Gauss law
$\phi=\frac{q_{\text {enclosed }}}{\varepsilon_{0}}=\frac{5 q}{\varepsilon_{0}}$
22. A stationary hydrogen atom de excites from first excited state to ground state. Find recoil speed of hydrogen atom up to nearest integral value. (mass of hydrogen atom $=1.8 \times 10^{-27} \mathrm{~kg}$ )
Answer (3)
Sol. $\left|\Delta E_{0}\right|=\left(-13.6\left\{1-\frac{1}{4}\right\}\right) \mathrm{ev}$
$|\Delta E|=10.2 \mathrm{ev}$

$$
\begin{aligned}
& \stackrel{v}{\longleftrightarrow} \longrightarrow \frac{h}{\lambda} \\
& \lambda=\frac{12400}{10.2} \times 10^{-10} \mathrm{~m} \\
& \rho=\frac{h}{\lambda}=\frac{6.63 \times 10^{-34} \times 10.2}{12400 \times 10^{-10}} \\
& \because m v=\frac{h}{\lambda} \\
& \therefore \quad 1.8 \times 10^{-27} \\
& v=\frac{6.63 \times 10.2 \times 10^{-34}}{12400 \times 10^{-10}} \\
& v=\frac{6.63 \times 10.2}{12400 \times 1.8} \times 10^{3} \\
& =\frac{6.63 \times 102}{124 \times 1.8}=3.02 \\
& \approx 3 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

23. In a container, 1 g of hydrogen and 1 g of oxygen are taken. Find the ratio of hydrogen pressure to oxygen pressure.

## Answer (16)

Sol. $P V=n R T$
$\Rightarrow P \propto n$
$\Rightarrow$ Ratio $=\frac{32}{2}=16$
24. In a convex mirror having radius of curvature 30 cm the height of image is half the object height. What will be the object (in cm ) distance?

## Answer (15)

Sol. $f=15$
$m=-\frac{v}{u}=+\frac{1}{2}$
$v=-\frac{u}{2}$
$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$
$\frac{2}{-u}+\frac{1}{u}=\frac{1}{f}$
$u=-f=-15 \mathrm{~cm}$
25. A solid cylinder is placed gently over an incline plane of inclination $60^{\circ}$. The acceleration of cylinder when it start rolling without slipping is $\frac{g}{\sqrt{x}}$, where $\mu$ is coefficient of friction. (Take $g=10 \mathrm{~m} / \mathrm{s}^{2}$ )

## Answer (3)

Sol.


Since $a=\frac{g \sin \theta}{1+\frac{l}{M R^{2}}}$

$$
\begin{aligned}
& \Rightarrow a=\frac{g \times \frac{\sqrt{3}}{2}}{1+\frac{1}{2}}=\frac{g \frac{\sqrt{3}}{2}}{\frac{3}{2}} \\
& \Rightarrow \quad a=\frac{g}{\sqrt{3}}
\end{aligned}
$$

26. Voltage and resistance for a resistor are measured as $V=200 \pm 5$ volts and $R=20 \pm 0.2 \Omega$. The percentage error in current $I=\frac{V}{R}$ is $x$. Find the value of $10 x$

## Answer (35)

Sol. \% error $=\left(\frac{d V}{V}+\frac{d R}{R}\right) \times 100$

$$
\begin{aligned}
& =\left(\frac{5}{200}+\frac{0.2}{20}\right) \times 100 \\
& =3.5
\end{aligned}
$$

27. Potential energy function corresponding to a conservative force is given as $U(x, y, z)=\frac{3 x^{2}}{2}+5 y+6 z$, then the force at $x=6$ is $p \mathrm{~N}$. The value of $p$ upto its nearest integral value is

## Answer (20)

Sol. $F_{x}=\frac{-d v}{d x}$

$$
\begin{aligned}
& \vec{F}=-3 x \hat{i}-5 \hat{j}-6 \hat{k} \\
& |\vec{F}|_{x=6}=\sqrt{18^{2}+5^{2}+6^{2}} \\
& =\sqrt{324+25+36} \\
& =\sqrt{385} \\
& =19.62 \mathrm{~N}
\end{aligned}
$$

28. 
29. 
30. 

## CHEMISTRY

## SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which ONLY ONE is correct.

## Choose the correct answer :

1. Which of the following pair will be formed by the decomposition of $\mathrm{KMnO}_{4}$ ?
(1) $\mathrm{KMnO}_{4}, \mathrm{MnO}_{2}$
(2) $\mathrm{K}_{2} \mathrm{MnO}_{4}, \mathrm{MnO}_{2}$
(3) $\mathrm{K}_{2} \mathrm{MnO}_{4}, \mathrm{H}_{2} \mathrm{O}$
(4) $\mathrm{MnO}_{2}, \mathrm{H}_{2} \mathrm{O}$

Answer (2)
Sol. $\mathrm{KMnO}_{4}$ decomposes upon heating at 513 K and forms $\mathrm{K}_{2} \mathrm{MnO}_{4}$ and $\mathrm{MnO}_{2}$.

$$
2 \mathrm{KMnO}_{4} \xrightarrow{\Delta} \mathrm{~K}_{2} \mathrm{MnO}_{4}+\mathrm{O}_{2}+\mathrm{MnO}_{2}
$$

2. 



In the following reactions, find the product A and B ?
(1)

(A)

(B)
(2)

(A)

(B)
(3)

(A)

(B)
(4)

(A)

(B)

## Answer (2)

Sol. - In presence of light allylic substitution occur.

- In presence of $\mathrm{CCl}_{4}$, addition reaction will occur.

3. The major product formed in the following reaction is:


(1)

(2)

(3)

(4)


## Answer (3)

Sol. HBr adds to alkene in accordance with Markovnikov's rule

4. Which of the following coordination compounds has bridging carbonyl ligand?
(1) $\left[\mathrm{Mn}_{2}(\mathrm{CO})_{10}\right]$
(2) $\left[\mathrm{Co}_{2}(\mathrm{CO})_{8}\right]$
(3) $\left[\mathrm{Cr}(\mathrm{CO})_{6}\right]$
(4) $\left[\mathrm{Fe}(\mathrm{CO})_{5}\right]$

Answer (2)

Sol.


From structure it is clear $\left[\mathrm{Co}_{2}(\mathrm{CO})_{8}\right]$ has bridging carbonyl ligand.
5. Energy difference between actual structure of compound and most stable resonating structure having least energy is called:
(1) Heat of hydrogenation
(2) Resonance energy
(3) Heat of combustion
(4) Exchange energy

Answer (2)
Sol. Resonance energy is the energy difference between most stable resonating structure and actual structure.
6. What is the effect that occurs between lone pair and $\pi$-bond?
(1) Inductive
(2) Electromeric
(3) Resonance
(4) Hyperconjugation

Answer (3)
Sol. $\stackrel{\odot}{X}-Y=Z$


Above effect is called Resonance.
Correct answer is option (3).
7. Which of the following statement is incorrect?
(1) $\Delta G=0$ for reversible process
(2) $\Delta G<0$ for spontaneous process
(3) $\Delta G>0$ for spontaneous process
(4) $\Delta G>0$ for non-spontaneous process

## Answer (3)

Sol. For spontaneous process $\Delta \mathrm{G}<0$
For reversible process $\Delta G=0$
8. Alkaline $\mathrm{KMnO}_{4}$ oxidises lodide to a particular product $(A)$. Determine the oxidation state of lodine in compound $(A)$.
(1) +2
(2) +3
(3) +5
(4) +7

Answer (3)
Sol. Potassium permanganate in alkaline medium oxidise lodide to lodate.
$2 \mathrm{MnO}_{4}^{-}+\mathrm{H}_{2} \mathrm{O}+\mathrm{I}^{\ominus} \longrightarrow 2 \mathrm{MnO}_{2}+2 \mathrm{OH}^{\ominus}+\mathrm{IO}_{3}^{\ominus}$

Compound A is $\mathrm{IO}_{3}^{\ominus}$. Therefore, oxidation state of $l$ is +5 .
9. Find product P of the following reaction.

(i) $\mathrm{H}_{2} \mathrm{O} / \mathrm{H}^{+}$
(1)

(2)

(3)

(4)


## Answer (3)

Sol.

10. A container contains $1 \mathrm{~g} \mathrm{H}_{2}$ gas and $1 \mathrm{~g} \mathrm{O}_{2}$ gas, what is the ratio of partial pressure of $\mathrm{H}_{2}$ and $\mathrm{O}_{2}$ $\left(\frac{\mathrm{p}_{\mathrm{H}_{2}}}{\mathrm{p}_{\mathrm{O}_{2}}}\right)$ ?
(1) $16: 1$
(2) $8: 1$
(3) $4: 1$
(4) $1: 1$

Answer (1)
Sol. $\mathrm{p}_{\mathrm{H}_{2}}=\mathrm{P}_{\mathrm{T}} \chi_{\mathrm{H}_{2}} \quad\left(\mathrm{P}_{\mathrm{T}}=\right.$ total pressure $)$

$$
\begin{aligned}
\left(\chi_{\mathrm{H}_{2}}=\text { mole fraction of } \mathrm{H}_{2}\right) \\
\mathrm{p}_{\mathrm{O}_{2}}=\mathrm{P}_{\mathrm{T}} \chi_{\mathrm{O}_{2}} \quad\left(\chi_{\mathrm{O}_{2}}=\text { mole fraction of } \mathrm{O}_{2}\right)
\end{aligned}
$$

$\frac{\mathrm{p}_{\mathrm{H}_{2}}}{\mathrm{p}_{\mathrm{O}_{2}}}=\frac{\chi_{\mathrm{H}_{2}}}{\chi_{\mathrm{O}_{2}}}=\frac{\mathrm{n}_{\mathrm{H}_{2}}}{\mathrm{n}_{\mathrm{O}_{2}}}$
$\mathrm{n}_{\mathrm{H}_{2}}=\frac{1}{2} \mathrm{~mol}$
$\mathrm{n}_{\mathrm{O}_{2}}=\frac{1}{32}$
$\frac{\mathrm{p}_{\mathrm{H}_{2}}}{\mathrm{p}_{\mathrm{O}_{2}}}=\frac{1}{2 \times 1} \times 32$
$\frac{\mathrm{p}_{\mathrm{H}_{2}}}{\mathrm{p}_{\mathrm{O}_{2}}}=\frac{32}{2}=\frac{16}{1}$
11. Match the following.

|  | Column I <br> (Ores) |  | Column II <br> (Formula) |
| :--- | :--- | :--- | :--- |
| (A) | Fluorspar | (p) | $\mathrm{Al}_{2} \mathrm{O}_{3.2} \mathrm{H}_{2} \mathrm{O}$ |
| (B) | Cryolite | (q) | $\mathrm{CaF}_{2}$ |
| (C) | Bauxite | (r) | $\mathrm{MgCO}_{3} . \mathrm{CaCO}_{3}$ |
| (D) | Dolomite | (s) | $\mathrm{Na}_{3}\left[\mathrm{AlF}_{6}\right]$ |

(1) (A)-(s); (B)-(q); (C)-(r); D-(p)
(2) (A)-(q); (B)-(s); (C)-(p); D-(r)
(3) (A)-(p); (B)-(q); (C)-(s); D-(r)
(4) (A)-(q); (B)-(s); (C)-(r); D-(p)

Answer (2)
Sol. (A) Fluorspar - $\mathrm{CaF}_{2}$
(B) Cryolite - $\mathrm{Na}_{3}\left[\mathrm{AlF}_{6}\right]$
(C) Bauxite - $\mathrm{Al}_{2} \mathrm{O}_{3} \cdot 2 \mathrm{H}_{2} \mathrm{O}$
(D) Dolomite - $\mathrm{MgCO}_{3} \cdot \mathrm{CaCO}_{3}$
12. Which of the following element(s) is/are confirmed by appearance of blood red colour with $\mathrm{FeCl}_{3}$ in Lassaigne's test?
(1) Presence of S only
(2) Presence of $\mathrm{N} \& \mathrm{~S}$
(3) Presence of N only
(4) Presence of $P$ only

## Answer (2)

Sol. $\mathrm{Na}+\mathrm{C}+\mathrm{N}+\mathrm{S} \rightarrow \mathrm{NaSCN}$

13. Statement 1 : Electronegativity of group 14 elements decreases from Si to Pb .
Statement 2 : Group 14 has metals, metalloids and non-metals.
(1) Both Statements 1 and 2 are correct
(2) Both Statements 1 and 2 are incorrect
(3) Statement 1 is correct and Statement 2 is incorrect
(4) Statement 1 is incorrect and Statement 2 is correct

## Answer (4)

Sol. Electronegativity generally decreases as we move down the group but Pb has higher electronegativity than Sn .

- $\mathrm{C} \Rightarrow$ non-metal

Si and $\mathrm{Ge} \Rightarrow$ metalloids
Sn and $\mathrm{Pb} \Rightarrow$ metals
E.N. of $\mathrm{Sn}=1.8, \mathrm{~Pb}=1.9$
14. Hydrolysis of proteins gives which type of amino acid?
(1) $\alpha$-Amino acid
(2) $\beta$-Amino acid
(3) $\gamma$-Amino acid
(4) $\delta$-Amino acid

## Answer (1)

Sol. Proteins on hydrolysis gives $\alpha$-amino acid because $\alpha$-amino acids are building block of proteins. It is also fact that amino acids contain both $-\mathrm{NH}_{2}$ and -COOH group.
15. Statement 1 : Ionisation energy decreases in a period.

Statement 2 : In a period Z dominates over screening effect
(1) Both statements 1 and 2 are correct
(2) Both statements 1 and 2 are incorrect
(3) Statement 1 is correct and statement 2 is incorrect
(4) Statement 1 is incorrect but statement 2 is correct

## Answer (4)

Sol. Ionisation enthalpy increases in a period. Z dominates over screening effect $(\sigma)$ in a period as $Z_{\text {eff. }}$ increases.
16. Consider the following reaction

(1)

(2)

(3) Both (1) \& (2)
(4) None of these

## Answer (2)

Sol.



17. Match the following

|  | Column I <br> (Complexes) |  | Column II <br> (Metals) |
| :--- | :--- | :--- | :--- |
| A. | Vitamin $\mathrm{B}_{12}$ | (p) | Ti |
| B. | Wilkinson catalyst | (q) | Co |
| C. | Ziegler-Natta catalyst | (r) | Fe |
| D. | Haemoglobin | (s) | Rh |

(1) $A(q), B(s), C(p), D(r)(2) A(s), B(q), C(r), D(p)$
(3) $A(q), B(p), C(r), D(s)$
(4) $A(q), B(r), C(p), D(s)$

Answer (1)
Sol. A. Vitamin $B_{12}$ - Co
B. Wilkinson catalyst $-\mathrm{Rh}\left(\left[\mathrm{Rh}\left(\mathrm{PPh}_{3}\right)_{3} \mathrm{Cl}\right]\right)$
C. Ziegler-Natta catalyst $-\mathrm{Ti}\left(\mathrm{TiCl}_{4}+\mathrm{Al}\left(\mathrm{C}_{2} \mathrm{H}_{5}\right)_{3}\right)$
D. Haemoglobin -Fe
18. $\mathrm{K}_{2} \mathrm{Cr}_{2} \mathrm{O}_{7}+\mathrm{H}_{2} \mathrm{O}_{2}+\mathrm{H}_{2} \mathrm{SO}_{4} \xrightarrow[\text { conditions }]{\text { ether cold }}$ compound ' $X$ ' X is a chromium compound, what is the oxidation state of chromium in compound ' $X$ '.
(1) +6
(2) +3
(3) +5
(4) +10

Answer (1)
Sol. $\mathrm{K}_{2} \mathrm{Cr}_{2} \mathrm{O}_{7}+\mathrm{H}_{2} \mathrm{O}_{2}+\mathrm{H}_{2} \mathrm{SO}_{4} \rightarrow \underset{\left(X^{\prime}\right)}{\mathrm{CrO}_{5}}+\mathrm{K}_{2} \mathrm{SO}_{4}+\mathrm{H}_{2} \mathrm{O}$
compound ' $X$ ' is $\Rightarrow \mathrm{CrO}_{5}$


Oxidation state of chromium $=+6$.
19. $\mathrm{xCl}_{2}+\mathrm{yOH}^{-} \longrightarrow \mathrm{ZCl}^{-}+\mathrm{pClO}^{-}$

Balance the above reaction and find out values of $x, y, z$ and $p$.
(1) $x=1, y=2, z=2, p=1$
(2) $x=y=z=p=1$
(3) $x=1, y=1, z=2, p=1$
(4) $x=1, y=2, z=1, p=1$

Answer (4)
Sol.


After balancing change in oxidation state,
$2 \mathrm{Cl}_{2} \longrightarrow 2 \mathrm{Cl}^{-}+2 \mathrm{ClO}^{-}$
Next, balance 'O’ atoms,
$2 \mathrm{Cl}_{2}+4 \mathrm{OH}^{-} \longrightarrow 2 \mathrm{Cl}^{-}+2 \mathrm{ClO}^{-}+2 \mathrm{H}_{2} \mathrm{O}$

Simplifying to get simplest ratios,
$\mathrm{Cl}_{2}+2 \mathrm{OH}^{-} \longrightarrow \mathrm{Cl}^{-}+\mathrm{ClO}^{-}+\mathrm{H}_{2} \mathrm{O}$
$x=1, y=2, z=1, p=1$
20. For $\mathrm{Rb}(37)$ which of the following set of quantum numbers are correct for valence electron?
(1) $5,0,0,+\frac{1}{2}$
(2) $5,0,1,-\frac{1}{2}$
(3) $5,0,1,+\frac{1}{2}$
(4) $5,1,1,+\frac{1}{2}$

Answer (1)
Sol. ${ }_{37} \mathrm{Rb}=1 s^{2} 2 s^{2} 2 p^{6} 3 s^{2} 3 p^{6} 3 d^{10} 4 s^{2} 4 p^{6} 5 s^{1}$
Last electron enters in $5 s$ subshell
Value of quantum numbers
$\mathrm{n}=5, \mathrm{l}=0, \mathrm{~m}=0, \mathrm{~s}= \pm \frac{1}{2}$

## SECTION - B

Numerical Value Type Questions: This section contains 10 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.
21. Calculate the molarity of a solution having density $=1.5 \mathrm{~g} / \mathrm{mL} \%(\mathrm{w} / \mathrm{w})$ of solute is $36 \%$ and molecular weight of solute is $36 \mathrm{~g} / \mathrm{mol}$.

## Answer (15)

Sol. Assume mass of solution

$$
=100 \mathrm{~g}
$$

Mass of solute $=36 \mathrm{gm}$
Moles of solute $=1$
Molarity $=\frac{1 \times 1000}{\left(\frac{100}{1.5}\right)}=\frac{1000}{100} \times 1.5=15$
22. Given $\mathrm{K}_{\mathrm{net}}=\frac{\mathrm{K}_{1} \mathrm{~K}_{2}}{\mathrm{~K}_{3}}$ when $\mathrm{E}_{\mathrm{a}_{1}}=40 \mathrm{~kJ} / \mathrm{mol}$
$\mathrm{E}_{\mathrm{a}_{2}}=50 \mathrm{~kJ} / \mathrm{mol}, \mathrm{E}_{\mathrm{a}_{3}}=60 \mathrm{~kJ} / \mathrm{mol}$.
Calculate value of $\left(\mathrm{E}_{\mathrm{a}}\right)_{\text {net }}$ in $\mathrm{kJ} / \mathrm{mol}$

## Answer (30)

Sol. $\left(E_{a}\right)_{n e t}=E_{a_{1}}+E_{a_{2}}-E_{a_{3}}$

$$
\begin{aligned}
& =40+50-60 \\
& =90-60 \\
& =30 \mathrm{~kJ} / \mathrm{mol}
\end{aligned}
$$

23. Positive Fehling solution test is given by


Answer (3)

Sol. Fehling solution test can be given by aldehyde except aromatic aldehyde.

other all three given can give Fehling solution test.
24. How many of the following compounds have one lone pair in central atom?
$\mathrm{ClF}_{3}, \mathrm{XeO}_{3}, \mathrm{BrF}_{5}, \mathrm{XeF}_{4}, \mathrm{O}_{3}, \mathrm{NH}_{3}$

## Answer (4)

Sol.

$\mathrm{lp}=2$

$\mathrm{lp}=1$


lp=2


25. How many of the following species have bond order $=1$ and are paramagnetic as well?
$\mathrm{He}_{2}^{2+} ; \mathrm{O}_{2}^{2-} ; \mathrm{Ne}_{2}^{2+} ; \mathrm{F}_{2} ; \mathrm{B}_{2} ; \mathrm{H}_{2} ; \mathrm{O}_{2}^{2+}$
Answer (1)
Sol. B2 have bond order equal to 1 and also paramagnetic.
$\mathrm{He}_{2}^{2+} ; \mathrm{O}_{2}^{2-} ; \mathrm{Ne}_{2}^{2+} ; \mathrm{F}_{2} ; \mathrm{H}_{2}$ have bond order equal to 1 but are diamagnetic.
$\mathrm{O}_{2}^{2+}$ have bond order equal to 3 .
26. How many of the following compound contain sulphur atom?
Pyrrole, Furan, Thiophene, Cysteine, Tyrosine, Pyridine

## Answer (2)

Sol.






Thiophene and cysteine contain sulphur atom.
27. Through a $\mathrm{ZnSO}_{4}$ solution, 0.015 A current was passed for 15 minutes. What is the mass of Zn deposited? (in mg)
(Atomic weight of $\mathrm{Zn}=65.4$ )

## Answer (5)

Sol. Charge passed $=$ It

$$
=0.015 \times 15 \times 60 \mathrm{C}
$$

Moles of electrons passed $=\frac{0.015 \times 15 \times 60}{96500}$
Moles of Zn deposited $=\frac{1}{2} \times \frac{0.015 \times 15 \times 60}{96500}$

$$
=0.00007
$$

Mass of Zn deposited $=0.00007 \times 65.4 \mathrm{~g}=4.58 \mathrm{mg}$
28. Osmotic pressure at 273 K is $7 \times 10^{5} \mathrm{~Pa}$, then what will be the value of $x$, if its osmotic pressure at 283 K is $\mathrm{x} \times 10^{4} \mathrm{~Pa}$ ?

Answer (73)
Sol. $\pi_{1}=$ iCRT $_{1}$
$\pi_{2}=\mathrm{iCRT}_{2}$
$\frac{\pi_{1}}{T_{1}}=\frac{\pi_{2}}{T_{2}}$
$\pi_{2}=\frac{\pi_{1} \times T_{2}}{T_{1}}$

$$
=\frac{7 \times 10^{5} \times 283}{273}
$$

$$
=7.256 \times 10^{5} \mathrm{~Pa}
$$

$$
=72.56 \times 10^{4} \mathrm{~Pa}
$$

$\pi_{2}=x \times 10^{4}$
$\therefore \quad x=72.56 \approx 73$
29. $K_{p}$ for the given reaction is $\left(36 \times 10^{-2} \mathrm{~atm}^{-1}\right)$. Find out $\mathrm{K}_{\mathrm{c}}\left(\mathrm{M}^{-1}\right)$ (nearest integer).
$\left(2 \mathrm{NO}_{2} \rightleftharpoons \mathrm{~N}_{2} \mathrm{O}_{4}\right)$
( $\mathrm{R}=0.0821 \mathrm{~atm} . \mathrm{L} / \mathrm{mol} . \mathrm{K}$ )
( $\mathrm{T}=300 \mathrm{~K}$ )
Answer (9)
Sol. $K_{p}=K_{c}(R T)^{\Delta n g}$
$36 \times 10^{-2}=K_{c}(0.0821 \times 300)^{-1}$
$\mathrm{K}_{\mathrm{c}}=0.36 \times 0.0821 \times 300=8.86 \approx 9$
30. ??

## MATHEMATICS

## SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which ONLY ONE is correct.

## Choose the correct answer :

1. Let a die rolled till 2 is obtained. The probability that 2 obtained on even numbered toss is equal to
(1) $\frac{5}{11}$
(2) $\frac{5}{6}$
(3) $\frac{1}{11}$
(4) $\frac{6}{11}$

## Answer (1)

Sol. $P(2$ obtained on even numbered toss $)=k$ (let)

$$
\begin{aligned}
& P(2)=\frac{1}{6} \\
& P(\overline{2})=\frac{5}{6} \\
& k=\frac{5}{6} \times \frac{1}{6}+\left(\frac{5}{6}\right)^{3} \times \frac{1}{6}+\left(\frac{5}{6}\right)^{5} \times \frac{1}{6}+\ldots \\
& =\frac{\frac{5}{6} \times \frac{1}{6}}{1-\left(\frac{5}{6}\right)^{2}} \\
& =\frac{5}{11}
\end{aligned}
$$

2. $\lim _{x \rightarrow \frac{\pi^{-}}{2}} \frac{\int_{x^{3}}^{\left(\frac{\pi}{2}\right)^{2}} \cos t^{1 / 3} d t}{\left(x-\frac{\pi}{2}\right)^{2}}$
(1) $\frac{3 \pi^{2}}{4}$
(2) $\frac{3 \pi}{4}$
(3) $\frac{3 \pi^{2}}{8}$
(4) $\frac{3 \pi}{8}$

## Answer (3)

Sol.
$\lim _{h \rightarrow 0} \frac{\int_{\left(\frac{\pi}{2}\right)^{3}}^{\left(\frac{\pi}{2}\right)^{3}} \cos \left(t^{1 / 3}\right) d t}{h^{2}}$
$=\lim _{h \rightarrow 0} \frac{0+3\left(\frac{\pi}{2}-h\right)^{2} \cos \left(\frac{\pi}{2}-h\right)}{2 h}$
$=\lim _{h \rightarrow 0} \frac{3\left(\frac{\pi}{2}-h\right)^{2} \sin h}{2 h}$
$=\frac{3 \pi^{2}}{8}$
3. Consider the equation $4 \sqrt{2} x^{3}-3 \sqrt{2} x-1=0$.

Statement 1: Solution of this equation is $\cos \frac{\pi}{12}$.
Statement 2: This equation has only one real solution.
(1) Both statement 1 and statement 2 are true
(2) Statement 1 is true but statement 2 is false
(3) Statement 1 is false but statement 2 is true
(4) Both statement 1 and statement 2 are false

Answer (2)
Sol. $12 x=\pi$
$\Rightarrow 3 x=\frac{\pi}{4}$
$\cos 3 x=\frac{1}{\sqrt{2}}$
$\Rightarrow 4 \cos ^{3} x-3 \cos x=\frac{1}{\sqrt{2}}$
$\Rightarrow 4 \sqrt{2} \cos ^{3} x-3 \sqrt{2} \cos x-1=0$
$x=\frac{\pi}{12}$ is the solution of above equation.
$\therefore$ Statement 1 is true
$f(x)=4 \sqrt{2} x^{3}-3 \sqrt{2} x-1$
$f^{\prime}(x)=12 \sqrt{2} x^{2}-3 \sqrt{2}=0$
$\Rightarrow x= \pm \frac{1}{2}$
$f\left(-\frac{1}{2}\right)=-\frac{1}{\sqrt{2}}+\frac{3}{\sqrt{2}}-1=\sqrt{2}-1>0$
$f(0)=-1<0$
$\therefore$ one root lies in $\left(-\frac{1}{2}, 0\right)$, one root is $\cos \frac{\pi}{12}$ which is positive. As the coefficients are real, therefore all the roots must be real.
$\therefore$ Statement 2 is false.
4. If $|2 A|^{3}=2^{21}$
and $A=\left[\begin{array}{lll}1 & 0 & 0 \\ 0 & \alpha & \beta \\ 0 & \beta & \alpha\end{array}\right]$ then $\alpha$ is (if $\alpha, \beta \in \mathrm{I}$ )
(1) 5
(2) 3
(3) 9
(4) 17

## Answer (1)

Sol. $|2 A|=2^{7}$
$8|A|=2^{7}$
$|A|=2^{4}$
Now $|A|=\alpha^{2}-\beta^{2}=2^{4}$
$\alpha^{2}=16+\beta^{2}$
$\alpha^{2}-\beta^{2}=16$
$(\alpha-\beta)(\alpha+\beta)=16$
$\Rightarrow \alpha+\beta=8$ and
$\alpha-\beta=2$
$\Rightarrow \alpha=5$, and $\beta=3$
5. In a 64 terms GP if sum of total terms is seven times sum of odd terms, then common ratio is
(1) 3
(2) 4
(3) 5
(4) 6

## Answer (4)

Sol. $a, a r, a r^{2}, \ldots . . a r^{63}$

$$
\begin{aligned}
& a+a r+a r^{2}+\ldots .+a r^{63}=7\left[a+a r^{2}+a r^{4}+\ldots . .+a r^{2}\right] \\
& \frac{a\left(1-r^{64}\right)}{(1-r)}=7 \frac{a\left(1-r^{64}\right)}{\left(1-r^{2}\right)} \\
& 1+r=7 \\
& r=6
\end{aligned}
$$

6. If $\frac{d y}{d x}-\left(\frac{\sin 2 x}{1+\cos ^{2} x}\right) y=\frac{\sin x}{1+\cos ^{2} x}$ and $y(0)=0$ then $y\left(\frac{\pi}{2}\right)$ is
(1) -1
(2) 1
(3) 0
(4) 2

## Answer (2)

Sol. $\frac{d y}{d x}-\left(\frac{\sin 2 x}{1+\cos ^{2} x}\right) y=\frac{\sin x}{1+\cos ^{2} x}$
$I F=e^{-\int \frac{\sin 2 x d x}{1+\cos ^{2} x}}$
$=e^{\ln \left(1+\cos ^{2} x\right)}=\left(1+\cos ^{2} x\right)$
So, $y\left(1+\cos ^{2} x\right)=\int \frac{\sin x}{\left(1+\cos ^{2} x\right)} \cdot\left(1+\cos ^{2} x\right) d x$
$y\left(1+\cos ^{2} x\right)=-\cos x+c$
$\because y(0)=0$
$0=-1+c$
$\Rightarrow c=1$
$y=\frac{1-\cos x}{1+\cos ^{2} x}$
Now, $y\left(\frac{\pi}{2}\right)=1$
7. $4 \cos \theta+5 \sin \theta=1$

Then find $\tan \theta$, where $\theta \in\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$.
(1) $\frac{10-\sqrt{10}}{6}$
(2) $\frac{10-\sqrt{10}}{12}$
(3) $\frac{\sqrt{10}-10}{6}$
(4) $\frac{\sqrt{10}-10}{12}$

## Answer (4)

Sol. $16 \cos ^{2} \theta+25 \sin ^{2} \theta+40 \sin \theta \cos \theta=1$
$16+9 \sin ^{2} \theta+20 \sin 2 \theta=1$
$16+9\left(\frac{1-\cos 2 \theta}{2}\right)+20 \sin 2 \theta=1$
$\frac{-9}{2} \cos 2 \theta+20 \sin 2 \theta=\frac{-39}{2}$
$-9 \cos 2 \theta+40 \sin 2 \theta=-39$
$-9\left(\frac{1-\tan ^{2} \theta}{1+\tan ^{2} \theta}\right)+40\left(\frac{2 \tan \theta}{1+\tan ^{2} \theta}\right)=-39$
$48 \tan ^{2} \theta+80 \tan \theta+30=0$
$24 \tan ^{2} \theta+40 \tan \theta+15=0$
$\tan \theta=\frac{-40 \pm \sqrt{(40)^{2}-15 \times 24 \times 4}}{2 \times 24}$
$\tan \theta=\frac{-40 \pm \sqrt{160}}{2 \times 24}$
$=\frac{-10 \pm \sqrt{10}}{12}$
$\Rightarrow \tan \theta=\frac{\sqrt{10}-10}{12}, \quad \tan \theta=\frac{-\sqrt{10}-10}{12}$
So $\tan \theta=-\frac{\sqrt{10}-10}{12}$ will be rejected as $\theta \in\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Option (4) is correct.
8. In an increasing arithmetic progression $a_{1}, a_{2}, \ldots . a_{n}$ if $a_{6}=2$ and product of $a_{1}, a_{5}$ and $a_{4}$ is greatest, then the value of $d$ is equal to
(1) 1.6
(2) 1.8
(3) 0.6
(4) 2.0

## Answer (1)

Sol. First term =a
Common difference $=d$
Given: $a+5 d=2$
Product $(P)=\left(a_{1} a_{5} a_{4}\right)=a(a+4 d)(a+3 d)$
Using (1)
$P=(2-5 d)(2-d)(2-2 d)$
$\Rightarrow \quad \frac{d P}{d d}=(2-5 d)(2-d)(-2)+(2-5 d)(2-2 d)(-1)$ $+(-5)(2-d)(2-2 d)$
$=-2[(d-2)(5 d-2)+(d-1)(5 d-2)+5(d-1)(d$ -2)]
$=-2\left[5 d^{2}+4-12 d+5 d^{2}+2-7 d+5 d^{2}+10-15\right.$
d]
$=-2\left[15 d^{2}-34 d+16\right]$
$\Rightarrow d=\frac{8}{5}$ or $\frac{2}{3}$
at $\left(\frac{8}{5}\right)$, product attains maxima
$\Rightarrow d=1.6$
9. If relation $R:(a, b) R(c, d)$ is only if $a d-b c$ is divisible by $5(a, b, c, d \in Z)$ then $R$ is
(1) Reflexive
(2) Symmetric, Reflexive but not Transitive
(3) Reflexive, Transitive but not symmetric
(4) Equivalence relation

## Answer (2)

Sol. Reflexive : for $(a, b) R(a, b)$
$\Rightarrow a b-a b=0$ is divisible by 5.
So $(a, b) R(a, b) \forall a, b \in Z$
$\therefore \quad R$ is reflexive
Symmetric:
For $(a, b) R(c, d)$
If $a d-b c$ is divisible by 5 .
Then $b c-a d$ is also divisible by 5.
$\Rightarrow \quad(c, d) R(a, b) \forall a, b, c, d \in Z$
$\therefore \quad R$ is symmetric
Transitive :
If $(a, b) R(c, d) \Rightarrow a d-b c$ divisible by 5 and $(c, d) R(e, f) \Rightarrow c f-d e$ divisible by 5
$a d-b c=5 k_{1} \quad k_{1}$ and $k_{2}$ are integers
$c f-d e=5 k_{2}$
afd $-b c f=5 k_{1} f$
$b c f-b d e=5 k_{2} b$
$a f d-b d e=5\left(k_{1} f+k_{2} b\right)$
$d(a f-b e)=5\left(k_{1} f+k_{2} b\right)$
$\Rightarrow a f-b e$ is not divisible by 5 for every $a, b, c, d$, $e, f \in Z$.
$\therefore \quad R$ is not transitive
For e.g., take $a=1, b=2, c=5, d=5, e=2, f=2$
10. Let $f(x)= \begin{cases}2+2 x, & x \in(-1,0) \\ 1-\frac{x}{3}, & x \in[0,3)\end{cases}$
$g(x)=\left\{\begin{array}{cc}x, & x \in[0,1) \\ -x, & x \in(-3,0)\end{array}\right.$
The range of $f \circ g(x)$ is
(1) $[0,1]$
(2) $[-1,1]$
(3) $(0,1]$
(4) $(-1,1)$

Answer (3)
Sol. $f(x)= \begin{cases}2+2 x, & x \in(-1,0) \\ 1-\frac{x}{3}, & x \in[0,3)\end{cases}$
$g(x)=\left\{\begin{array}{cc}x, & x \in[0,1) \\ -x, & x \in(-3,0)\end{array} \Rightarrow g(x)=|x|, x \in(-3,1)\right.$
$f(g(x))=\left\{\begin{array}{cl}2+2|x|, & |x| \in(-1,0) \Rightarrow x \in \phi \\ 1-\frac{|x|}{3}, & |x| \in[0,3) \Rightarrow x \in(-3,1)\end{array}\right.$
$f(g(x))= \begin{cases}1-\frac{x}{3}, & x \in[0,1) \\ 1+\frac{x}{3}, & x \in(-3,0)\end{cases}$


Range of $f \circ g(x)$ is $[0,1]$
11. If $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left(\frac{x^{2} \cos x}{1+\pi^{x}}+\frac{1+\sin ^{2} x}{1+e^{(\sin x)^{2023}}}\right) d x=\frac{\pi}{4}(\pi+\alpha)-2$

Then the value of ' $\alpha$ ' is equal to
(1) 1
(2) 2
(3) 3
(4) 4

Answer (3)

Sol. Given

$$
\begin{align*}
& \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left(\frac{x^{2} \cos x}{1+\pi^{x}}+\frac{1+\sin ^{2} x}{1+e^{(\sin x)^{2023}}}\right) d x=\frac{\pi}{4}(\pi+\alpha)-2 \\
& \int_{0}^{\frac{\pi}{2}}\left\{\left(\frac{x^{2} \cos x}{1+\pi^{x}}+\frac{1+\sin ^{2} x}{1+e^{(\sin x)^{202 s}}}\right)+\left(\frac{x^{2} \cos x}{1+\pi^{-x}}+\frac{1+\sin ^{2} x}{1+e^{-(\sin x)^{202 s}}}\right)\right\} d x \\
& =\frac{\pi}{4}(\pi+\alpha)-2 \\
& \int_{0}^{\frac{\pi}{2}}\left(x^{2} \cos x+1+\sin ^{2} x\right) d x=\frac{\pi}{4}(\pi+\alpha)-2 \\
& \int_{0}^{\frac{\pi}{2}} x^{2} \cos x d x+\int_{0}^{\frac{\pi}{2}}\left(1+\sin ^{2} x\right) d x=\frac{\pi}{4}(\pi+\alpha)-2 \quad \ldots(1) \tag{1}
\end{align*}
$$

Let $I_{1}=\int_{0}^{\frac{\pi}{2}}\left(1+\sin ^{2} x\right) d x$
$I_{1}=\int_{0}^{\frac{\pi}{2}} 1 \cdot d x+\int_{0}^{\frac{\pi}{2}}\left(\frac{1-\cos 2 x}{2}\right) d x$
$I_{1}=\frac{\pi}{2}+\frac{1}{2}\left[\frac{\pi}{2}+0\right]$
$I_{1}=\frac{\pi}{2}+\frac{\pi}{4}$
$l_{1}=\frac{3 \pi}{4}$
Let $I_{2}=\int_{0}^{\frac{\pi}{2}} x^{2} \cos x d x$
$I_{2}=\left[x^{2}(\sin x)-\int 2 x \int \cos x d x\right]_{0}^{\frac{\pi}{2}}$
$I_{2}=\left[x^{2}(\sin x)-2 \int x \sin x\right]_{0}^{\frac{\pi}{2}}$
$I_{2}=\left[x^{2} \sin x-2\left(x(-\cos x)+\int \cos x\right)\right]_{0}^{\frac{\pi}{2}}$
$I_{2}=\left[x^{2} \sin x-2(-x \cos x+\sin x)\right]_{0}^{\frac{\pi}{2}}$
$I_{2}=\left(\frac{\pi^{2}}{4}-2\right)$
$\therefore$ Put $l_{1}$ and $l_{2}$ in (1)
$\therefore \quad \frac{\pi^{2}}{4}-2+\frac{3 \pi}{4}$
$\frac{\pi^{2}}{4}+\frac{3 \pi}{4}-2$

$$
\frac{\pi}{4}(\pi+3)-2
$$

$\therefore \quad \alpha=3$
12. Area under the curve $x^{2}+y^{2}=169$ and below the line $5 x-y=13$ is
(1) $\frac{169 \pi}{4}-\frac{65}{2}+\frac{169}{2} \sin ^{-1} \frac{12}{13}$
(2) $\frac{169 \pi}{4}+\frac{65}{2}-\frac{169}{2} \sin ^{-1} \frac{12}{13}$
(3) $\frac{169}{4}-\frac{65}{2}+\frac{169}{2} \sin ^{-1} \frac{13}{14}$
(4) $\frac{169 \pi}{4}+\frac{65}{2}+\frac{169}{2} \sin ^{-1} \frac{13}{14}$

## Answer (1)

Sol.


$$
\begin{aligned}
& \text { Area }=\frac{\pi(13)^{2}}{2}-\left[\frac{1}{2} \times 25 \times 5+\int_{12}^{13} \sqrt{\left(169-y^{2}\right)} \cdot d y\right] \\
& =\frac{169 \pi}{2}-\left[\frac{125}{2}+\left[\frac{y}{2} \sqrt{169-y^{2}}+\frac{169}{2} \sin ^{-1} \frac{y}{13}\right]_{12}^{13}\right] \\
& =\frac{169}{2} \pi-\frac{125}{2}-\left[\frac{169}{2} \times \frac{\pi}{2}-6 \times 5-\frac{169}{2} \sin ^{-1} \frac{12}{13}\right] \\
& =\frac{169 \pi}{4}-\frac{65}{2}+\frac{169}{2} \sin ^{-1} \frac{12}{13}
\end{aligned}
$$

13. If $f(x)=\frac{\left(2^{x}+2^{-x}\right)(\tan x) \sqrt{\tan ^{-1}\left(2 x^{2}-3 x+1\right)}}{\left(7 x^{2}-3 x+1\right)^{3}}$, then $f(0)$ is equal to
(1) $\sqrt{\pi}$
(2) $\sqrt{\frac{\pi}{4}}$
(3) $\pi$
(4) $2 \cdot \pi^{3 / 2}$

Answer (1)

Sol. $f(x)=\frac{\left(2^{x}+2^{-x}\right) \tan x \sqrt{\tan ^{-1}\left(2 x^{2}-3 x+1\right)}}{\left(7 x^{2}-3 x+1\right)^{3}}$
$f(x)=\left(2^{x}+2^{-x}\right) \cdot \tan x \cdot \sqrt{\tan ^{-1}\left(2 x^{2}-3 x+1\right)} \cdot\left(7 x^{2}-3 x+1\right)^{-3}$
$f^{\prime}(x)=\left(2^{x}+2^{-x}\right) \cdot \sec ^{2} x \cdot \sqrt{\tan ^{-1}\left(2 x^{2}-3 x+1\right)} \cdot\left(7 x^{2}-3 x+1\right)^{-3}+\tan x \cdot(Q(x))$
$\therefore f^{\prime}(0)=2 \cdot 1 \cdot \sqrt{\frac{\pi}{4}} .1$
$=\sqrt{\pi}$
14. $\int \frac{(\sin x-\cos x) \sin ^{2} x}{\sin x \cos ^{2} x+\tan x \sin ^{3} x} d x$ is equal to
(1) $\frac{\ln \left|\sin ^{3} x-\cos ^{3} x\right|}{3}+c$
(2) $\frac{\ln \left|\sin ^{3} x+\cos ^{3} x\right|}{3}+c$
(3) $\frac{\ln \left|\sin ^{3} x-\cos ^{3} x\right|}{2}+c$
(4) $\frac{\ln \left|\sin ^{3} x+\cos ^{3} x\right|}{4}+c$

## Answer (2)

Sol. $\int \frac{(\sin x-\cos x) \sin ^{2} x}{\tan x\left(\sin ^{3} x+\cos ^{3} x\right)} d x$

$$
\int \frac{(\sin x-\cos x) \sin x \cos x}{\sin ^{3} x+\cos ^{3} x} d x, \text { put } \sin ^{3} x+\cos ^{3} x=t
$$

$$
\left(3 \sin ^{2} x \cdot \cos x-3 \cos ^{2} x \sin x\right) d x=d t
$$

$$
\Rightarrow \quad \frac{1}{3} \int \frac{d t}{t}
$$

$$
=\frac{\ln t}{3}+c
$$

$$
=\frac{\ln \left|\sin ^{3} x+\cos ^{3} x\right|}{3}+c
$$

15. 
16. 
17. 
18. 
19. 
20. 

## SECTION - B

Numerical Value Type Questions: This section contains 10 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.
21. $\frac{{ }^{11} C_{1}}{2}+\frac{{ }^{11} C_{2}}{3}+\ldots .+\frac{{ }^{11} C_{9}}{10}=\frac{m}{n}$

Then $m+n$ is
Answer (2041)
Sol. $(1+x)^{11}={ }^{11} C_{0}+{ }^{11} C_{1} x+{ }^{11} C_{2} x^{2}+\ldots .+{ }^{11} C_{11} x{ }^{11}$

$$
\begin{aligned}
& \begin{array}{l}
\int_{0}^{1}(1+x)^{11} d x={ }^{11} C_{0} x+ \\
+\frac{{ }^{11} C_{1} x^{2}}{2}+\frac{{ }^{11} C_{2} x^{3}}{3}+\ldots \\
\\
\left.+\frac{{ }^{11} C_{9} x{ }^{10}}{10}+\frac{{ }^{11} C_{10} x{ }^{11}}{11}+\frac{{ }^{11} C_{11} x}{12}\right]_{0}^{12} \\
\left.\frac{(1+x)^{12}}{12}\right]_{0}^{1}={ }^{11} C_{0}+\frac{{ }^{11} C_{1}}{2}+\frac{{ }^{11} C_{2}}{3}+\ldots .+\frac{{ }^{11} C_{9}}{10}+\frac{{ }^{11} C_{10}}{11}+\frac{{ }^{11} C_{11}}{12} \\
\frac{2^{12}-1}{12}-1-1-\frac{1}{12}=\frac{{ }^{11} C_{1}}{2}+\frac{{ }^{11} C_{2}}{3}+\ldots+\frac{{ }^{11} C_{10}}{11} \\
= \\
\frac{2^{12}-2-24}{12} \\
= \\
m+n=2035+6=2041
\end{array} \\
& m+\frac{2^{12}-26}{12}=\frac{4070}{12}=\frac{2035}{6}=\frac{m}{n}
\end{aligned}
$$

22. Rank of the word 'GTWENTY' in dictionary is

Answer (553)
Sol. Start with
(1) $\bar{E}: \frac{6!}{2!}=360$
(2) $\overline{G E}: \frac{5!}{2!}, \overline{G N}: \frac{5!}{2!}$
(3) GTE : 4!, GTN: 4!, GTT : 4!
(4) GTWENTY = 1
$\Rightarrow 360+60+60+24+24+24+1=553$
23. Curve $y=2^{x}-x^{2}, y(x) \& y^{\prime}(x)$ cut $x$-axis in $M \& N$ number of points respectively, find $M+N$.

## Answer (5)

Sol. $y(x)=2^{x}-x^{2}$

$$
y^{\prime}(x)=2^{x} \log 2-2 x
$$


$M=3$
$N=2$
$M+N=5$

Aakash
24. Given data
$60,60,44,58,68, \alpha, \beta, 56$ has mean 58 , variance $=66.2$ then find $\alpha^{2}+\beta^{2}$

## Answer (7182)

Sol. Variance $=\frac{\Sigma x^{2}}{n}-(\bar{x})^{2}$

$$
\begin{aligned}
& \begin{array}{l}
\frac{60^{2}+60^{2}+44^{2}+58^{2}+68^{2}+\alpha^{2}+\beta^{2}+56^{2}}{8} \\
-(58)^{2}=66.2
\end{array} \\
& \begin{array}{r}
7200+1936+3364+4624+3136+\alpha^{2}+\beta^{2} \\
8
\end{array}-3364=66.2 \\
& 2532.5+\frac{\alpha^{2}+\beta^{2}}{8}-3364=66.2 \\
& \alpha^{2}+\beta^{2}=897.7 \times 8 \\
& =7181.6
\end{aligned}
$$

25. If $|z+1|=\alpha z+\beta(i+1)$ and $z=\frac{1}{2}-2 i$, find $\alpha+\beta$.

## Answer (3)

Sol. $\left|\frac{1}{2}-2 i+1\right|=\alpha\left(\frac{1}{2}-2 i\right)+\beta(1+i)$
$\sqrt{\frac{9}{4}+4}=\alpha\left(\frac{1}{2}-2 i\right)+\beta(1+i)$
$\frac{5}{2}=\alpha\left(\frac{1}{2}\right)+\beta+i(-2 \alpha+\beta)$
$\frac{\alpha}{2}+\beta=\frac{5}{2}$
$-2 \alpha+\beta=0$
Solving (1) and (2)
$\frac{\alpha}{2}+2 \alpha=\frac{5}{2}$
$\frac{5}{2} \alpha=\frac{5}{2}$
$\alpha=1$
$\beta=2$
$\Rightarrow \alpha+\beta=3$
26. If $\vec{a}, \vec{b}, \vec{c}$ are non-zero and $\vec{b}$ and $\vec{c}$ are noncollinear. $\vec{a}+5 \vec{b}$ is collinear with $\vec{c}$ and $\vec{b}+6 \vec{c}$ is collinear with $\vec{a}$. If $\vec{a}+\alpha \vec{b}+\beta \vec{c}=0$, then find $\alpha+\beta$.

## Answer (35)

Sol. $\because \vec{a}+5 \vec{b}$ is collinear with $\vec{c}$

$$
\begin{equation*}
\Rightarrow \vec{a}+5 \vec{b}=\lambda \vec{c} \tag{1}
\end{equation*}
$$

$\vec{b}+6 \vec{c}$ is collinear with $\vec{a}$

$$
\begin{equation*}
\Rightarrow \vec{b}+6 \vec{c}=\mu \vec{a} \tag{2}
\end{equation*}
$$

From (1) and (2)
$\vec{b}+6 \vec{c}=\mu(\lambda \vec{c}-5 \vec{b})$
$\Rightarrow(1+5 \mu) \vec{b}+(6-\lambda \mu) \vec{c}=0$
$\because \vec{b}$ and $\vec{c}$ are non-collinear
$\Rightarrow 1+5 \mu=0 \Rightarrow \mu=\frac{-1}{5}$ and
$6-\lambda \mu=0 \Rightarrow \lambda \mu=6$
$\Rightarrow \lambda=-30$
Now,

$$
\vec{b}+6 \vec{c}=\frac{-1}{5} \vec{a}
$$

$$
5 \vec{b}+30 \vec{c}=-\vec{a}
$$

$$
\vec{a}+5 \vec{b}+30 \vec{c}=0
$$

$$
\vec{a}+\alpha \vec{b}+\beta \vec{c}=0
$$

On comparing

$$
\alpha=5, \beta=30 \Rightarrow \alpha+\beta=35
$$

27. 
28. 
29. 
30. 
